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INFLUENCE OF MEDIUM'S PECLET NUMBERS ON HEAT EXCHANGE IN FIBER BUNDLES

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Molding of synthetic fibers is carried out in both gas (melt molding, dry molding) and liquid media (wet and mixed molding). The wide spread in the physical parameters of the media in which bundles of molded fibers move, makes the question of the influence of these parameters on the intensity of heat and mass exchange very important.

For an isolated fiber, this problem can be solved numerically subject to the corresponding boundary conditions. Comprehensive analysis of the influence of the thermophysical parameters of the medium (the Prandtl number Pr) on the heat exchange of streamlined surfaces has been performed (see, for example, [1, 2]) by numerical and approximate methods within the framework of boundary-layer theory. As a result, some mechanisms and relations were established. Clear-cut formulation of the problem within the framework of the Navier–Stokes or boundary-layer models is possible only for bundles with specific geometrical arrangements of fibers. However, in the general case of movement of a molded bundle, there are no simple regular geometrical schemes for fiber arrangement. In this connection, a more general concept, namely, the model of filtration flow in a porous body, is used in such systems to describe transfer processes. This approach to numerical modeling of heat exchange in molded bundles has been developed by the authors in a number of papers (in particular, in [3, 4], where the basic equations and boundary conditions are formulated and the results of numerical analysis of particular variants are given).

An important feature of the filtration flow model is the uncertainty of the quantities of the dynamic and heat- and mass-exchange interactions of the flow with a fiber (an element of the dispersed medium). The method of cells [5, 6] is widely used in the mechanics of heterogeneous media to determine analytically these parameters. Using this method, a specific problem is solved in a region around the element subject to the boundary conditions on the inner surface motivated by physical considerations. Conditions on the outer boundary of the cell are specified by authors in accordance with their assumptions. For closure of the dynamic and heat-exchange problem, Eliseev and Sovit [3, 4] proposed integral conditions that relate the interaction parameters to the local filtration parameters of flow.

In this paper, conditions of validity for the previous relations are determined, an approximate computation method is presented, and heat-exchange parameters at various Peclet numbers Pe are found.

1. Heat Exchange in Cell. To find the heat flux on the surface of a moving fiber, we write a boundary-layer equation in the axisymmetric system of the x and r coordinates:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=-\frac{dp}{\rho dx}+\nu\frac{\partial}{r\partial r}\Big(r\frac{\partial u}{\partial r}\Big),\quad \frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial r}=0,$$
$$\rho c_p\Big(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}\Big)=\lambda\frac{\partial}{r\partial r}\Big(r\frac{\partial T}{\partial r}\Big). \tag{1.1}$$

Here u and v are velocity-vector components that correspond to the x and r axes; p is the pressure; ρ is the density; T is the temperature; c_p is the specific heat, ν is the kinematic viscosity; and λ is the heat

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conductivity. The boundary conditions on the surface of the fiber

$$u(r_n) = U_n, \quad T(r_n) = T_n \tag{1.2}$$

(U_n and T_n are the velocity and temperature of the fiber) and the integral conditions

$$2\pi \int_{r_n}^{r_\Delta} ru\,dr = \pi r_\Delta^2 u_f = \pi(r_\Delta^2 - r_n^2)U_m, \quad 2\pi \int_{r_n}^{r_\Delta} ruT\,dr = \pi r_\Delta^2 u_f T_f = \pi(r_\Delta^2 - r_n^2)U_m T_m \tag{1.3}$$

are the closure conditions for this system. Here r_n is the radius of the fiber; r_Δ is the outside radius of the cell; u_f is the filtration velocity of the medium, T_f is the filtration temperature of the medium; U_m is the mean velocity of the gas over the cell; and T_m is the mean calorimetric temperature.

Axisymmetric boundary-layer equations (1.1) used as a basis for consideration of heat exchange are two-dimensional and do not have exact analytical solutions for arbitrary U_n and T_n. Therefore, it is necessary to find approximate solutions or reduce (1.1) to one-dimensional differential equations consistent with the complete problem of heat exchange of a moving fiber bundle. An iterative method was used in [3, 4] to obtain simple analytical expressions, and the solution of the reduced equations of system (1.1) with discarded left-hand sides was used as a zeroth approximation:

$$u = U_n + \frac{1}{4}\frac{dp}{\mu dx}(r^2 - r_n^2) + \left[U_\Delta - U_n - \frac{1}{4}\frac{dp}{\mu dx}(r_\Delta^2 - r_n^2)\right]\frac{\ln(r/r_n)}{\ln(r_\Delta/r_n)},$$
$$T = T_n + (T_\Delta - T_n)\frac{\ln(r/r_n)}{\ln(r_\Delta/r_n)}, \tag{1.4}$$

where U_Δ and T_Δ are found from relations (1.3).

The iterative method is rather simple and gives analytical expressions for a number of problems, and, therefore, it is useful for obtaining approximate solutions and estimates. It has found application in boundary-layer theory ([7] is one of the first works) and can be successfully used for constant thermophysical parameters (the advantages and disadvantages of the method are discussed in [8, 9]). In our case, this method makes it possible to obtain a system of shape parameters similar to those used in the multiparameter method of [10] and, hence, to reveal the degree of nonequilibrium of the flow. According to the multiparameter method, the flow and heat exchange in the boundary layer are characterized by an infinite system of shape parameters, which generally determine the degree of nonequilibrium of the flow; therefore, the smaller the magnitude of these parameters, the closer the process to a stabilized process.

Expressions (1.4) are, strictly speaking, solutions of stabilized flow. They take into account parametrically variations of U_n and T_n, U_m, and T_m along the x axis, but do not take into account the influence of the longitudinal velocity and temperature gradients which cause nonequilibrium in flows. The next approximation can compensate for this shortcoming. However, the solutions found by the authors of this paper have shown that their use is not effective, because the accuracy is not high at $\mathrm{Pe} \gg 1$ and the analytical expressions are rather cumbersome. In view of this, we propose a method that allows one to obtain a simple analytical solution which combines zeroth and first approximations. Its idea is close to that in [11], where the derivative of temperature with respect to the longitudinal coordinate is replaced by a constant value. In our case, we substitute the mean values of $U_m(dT_m/dx)$ for $u(\partial T/\partial x)$ to obtain an analytical solution. Then, the solution of the equation

$$\lambda\frac{\partial}{r\partial r}\left(r\frac{\partial T}{\partial r}\right) = \rho c_p U_m \frac{dT_m}{dx}$$

has the form

$$T = \frac{\mathrm{Pe}}{4}(n^2 - n_*^2)\frac{dT_m}{d\xi} + T_n + \left[T_\Delta - T_n - \frac{\mathrm{Pe}}{4}(1 - n_*^2)\frac{dT_m}{d\xi}\right]\frac{\zeta}{\zeta_*}, \tag{1.5}$$

where $\xi = x/r_\Delta$; $n = r/r_\Delta$; $n_* = r_n/r_\Delta$; $\zeta = \ln(n/n_*)$; $\zeta_* = \ln(1/n_*)$; $\mathrm{Pe} = \mathrm{RePr}$; $\mathrm{Re} = r_\Delta U_m/\nu$; and $\mathrm{Pr} = \rho c_p \nu/\lambda$. We obtain the heat flux on the surface of the fiber q_n from (1.5) after determining T_Δ from the second condition of (1.3).

TABLE 1

ξ	Pe	q_T	q_p
	1	−24.85	−24.77
−2	10	−25.62	−25.14
	100	−27.06	−25.38
	1	−44.87	−44.31
0	10	−50.57	−47.05
	100	−61.25	−48.79
	1	−192.8	−188.7
2	10	−234.9	−208.9
	100	−313.8	−221.8

TABLE 2

ξ	Pe	q_T	q_p
	1	−167.6	−172.7
−2	10	−278.1	−280.6
	100	−262.8	−255.5
	1	−24.39	−24.48
2	10	−26.41	−26.46
	100	−26.13	−26.00

To obtain the accuracy of the solution above, let us consider a model problem of heat exchange in an infinite ring channel with stabilized liquid flow with the velocity profile

$$u = U_{\mathrm{m}}A\left[n^2 - n_*^2 - (1-n_*^2)\frac{\zeta}{\zeta_*}\right], \quad A = \left[(1-n_*^2) - \frac{2\zeta_* - (1-n_*^2)}{\zeta_*}\right]^{-1},$$

subject to zero conditions on the outer and inner walls. The boundary conditions for temperature are given by the simple expressions

$$T(1) = 0, \quad T(n_*) = 1 + \exp(æ\xi), \tag{1.6}$$

which make it possible to find an exact solution in a rather simple manner. It can be obtained numerically, if we assume that $T = T_n + T_{æ}\exp(æ\xi)$, where $T_n = 1 - \zeta_*^{-1}\zeta$, and $T_{æ}$ satisfies the following equation and the boundary conditions:

$$\frac{d^2T_{æ}}{dn^2} + \frac{dT_{æ}}{ndn} = æ\mathrm{Pe}A\left[n^2 - n_*^2 - (1-n_*^2)\frac{\zeta}{\zeta_*}\right]T_{æ}, \quad T_{æ}(n_*) = 1, \quad T_{æ}(1) = 0.$$

Comparison of the exact and approximate solutions given in Tables 1 ($æ = 1$) and 2 ($æ = -1$) shows that at Pe = 1 there is good agreement between values of $q_T = \partial T/\partial n_{n=n_*}$ (accurate solution) and $q_p = \partial T/\partial n_{n=n_*}$ (exact solution) for both $æ = 1$ and $æ = -1$. At Pe = 10, the agreement is also satisfactory, but at Pe = 100 for $æ = 1$, the difference becomes substantial. It should be noted, however, that in the model problem the longitudinal temperature gradients which are compared have a factor of exp (−2) to exp (2), while in reality it is less than 0.01 in the area with the most intense cooling. This suggests that the approximate solution found can be used to calculate the heat exchange of fibers in moving bundles.

2. Some Results of Analytical and Numerical Analyses. Let us consider a number of simple problems of theoretical and practical significance. We assume that the bundle is thick enough, i.e., in accordance with the solutions of [12], the direct influence of the boundary conditions on the surface of the bundle is shielded, and the flow is considered hydrodynamically stabilized. In this case, it is assumed that the heat exchange in the central zone of the bundle occurs in infinite bundles, and, hence, the equation of heat exchange obtained in [3, 4] is simplified and written as

$$\varepsilon\mathrm{Pe}\frac{dT_{\mathrm{f}}}{d\xi} = 2n_*\left[a(T_{\mathrm{n}} - T_{\mathrm{f}}) + b\mathrm{Pe}\frac{dT_{\mathrm{f}}}{d\xi}\right], \tag{2.1}$$

where ε is the bundle porosity; $a = (1-n_*^2)u_{\mathrm{f}}/(2n_*U_a)$; and the second term in the brackets is due to the filtration-temperature gradient;

$$b = \frac{1}{4n_*}\left(\frac{U_b}{U_a} - 2n_*^2\right); \quad U_a = \frac{1}{4}\frac{r_\Delta^2 dp}{\mu dx}i_1 + U_{\mathrm{n}}i_2 + \zeta_*^{-1}U_c i_3;$$

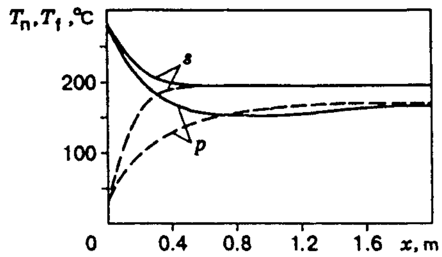

Fig. 1

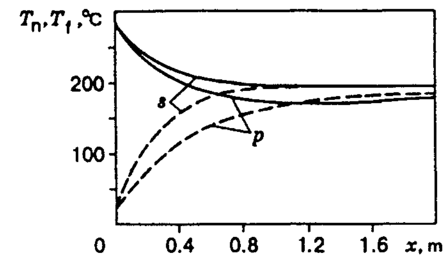

Fig. 2

$$U_b = \frac{1}{4}\frac{r_\Delta^2 dp}{\mu dx} i_4 + U_n i_5 + \zeta_*^{-1} U_c i_1; \quad U_c = \frac{2\zeta_*(1-n_*^2)}{2\zeta_* - 1 + n_*^2}\left(u_f - U_n - \frac{1}{4}\frac{r_\Delta^2 dp}{\mu dx}(1-n_*^2)\right);$$

$$i_1 = 0.25[(1-2n_*^2)\zeta_* - 0.25(1-3n_*^2)(1-n_*^2)]; \quad i_2 = 0.5[\zeta_* - 0.5(1-n_*^2)];$$

$$i_3 = 0.5[\zeta_*^2 - \zeta_* + 0.5(1-n_*^2)]; \quad i_4 = 0.166[(1-n_*^6) - 3n_*^2(1-n_*^4) - 3n_*^4(1-n_*^2)]; \quad i_5 = 0.25(1-n_*^2)^2.$$

Let us consider three cases: 1) the fibers have a constant temperature; 2) heat exchange with constant heat release from the fiber surface; and 3) mixed heat exchange.

In the first case, Eq. (2.1) can be rewritten as

$$\mathrm{Pe}(\varepsilon - 2n_* b)\frac{d(T_f - T_n)}{d\xi} = -2n_* a(T_f - T_n),$$

and then its solution contains the simple expression

$$T_f - T_n = B\exp(-\chi\xi), \quad \chi = \frac{2n_* a}{\mathrm{Pe}(\varepsilon - 2n_* b)}, \tag{2.2}$$

from which it follows that the Nusselt number

$$\mathrm{Nu}_n = \frac{n_*}{T_f - T_n}\frac{\partial T}{\partial n_{n=n_*}} = \frac{\varepsilon n_* a}{\varepsilon - 2n_* b} \tag{2.3}$$

does not depend on the Peclet number of the medium.

In the second case of constant heat release, we have the equations

$$\rho c_p u_f \frac{dT_f}{d\xi} = 2n_* q_n, \qquad q_n = \frac{\lambda}{r_\Delta}\left[a(T_n - T_f) + b\mathrm{Pe}\frac{dT_f}{d\xi}\right] = \mathrm{const} \tag{2.4}$$

with the simple solutions

$$T_f = 2\frac{n_* q_n}{\rho c_p u_f}\xi, \quad T_f - T_n = -\frac{r_\Delta}{\varepsilon\lambda a} q_n(\varepsilon - 2n_* b).$$

It follows from them that $\mathrm{Nu}_n = \varepsilon n_* a/(\varepsilon - 2n_* b)$ i.e., the Nusselt number coincides with expression (2.3).

In the third case, to Eq. (2.1) we add the equation of heat exchange for a fiber, which is written as

$$\mathrm{Pe}_n\frac{dT_n}{d\xi} = 2\frac{\lambda}{\lambda_n}\left[a(T_n - T_f) + b\mathrm{Pe}\frac{dT_f}{d\xi}\right], \tag{2.5}$$

where $\mathrm{Pe}_n = \rho_n c_{pn} r_n U_n/\lambda_n$; ρ_n is the material density; c_{pn} is the specific heat of the material; and λ_n is the heat conductivity in the fiber. After simple transformations, (2.1) and (2.5) can be reduced to the equation

$$\mathrm{Pe}(\varepsilon - 2bS_*)\frac{d(T_f - T_n)}{d\xi} = 2aS_*(T_f - T_n) \quad \left(S_* = n_* + \frac{n_*(\varepsilon - 2n_* b)\,\mathrm{Pe}}{\mathrm{Pe}_n + 2(\lambda/\lambda_n)n_* b\mathrm{Pe}}\frac{\lambda}{\lambda_n}\right).$$

Using solutions of the type of (2.2), we obtain $\mathrm{Nu} = \varepsilon n_* a/(\varepsilon - 2S_* b)$. This expression is similar in form to formula (2.3), but contains Peclet numbers for flow in the interfiber space and for a moving fiber.

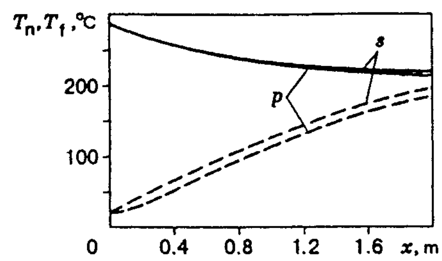

Fig. 3

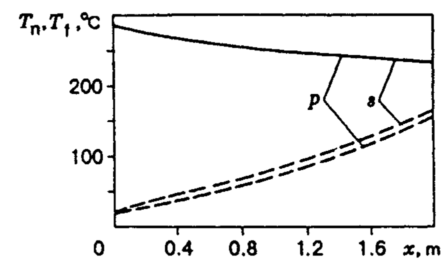

Fig. 4

Let us consider the question of how the Prandtl number of the medium affects the heat exchange of fiber bundles moving in a tube. Figures 1–4 give calculation results of the model problems for Pr = 0.5, 1, 5, and 10, respectively. The following values were used in the calculations: radius of the tube $R_t = 0.1$ m, radius of the bundle $R_b = 0.05$ m, fiber radius $r_n = 0.125 \cdot 10^{-3}$ m, fiber velocity $U_n = 0.3$ m/sec, number of fibers $N = 100$, initial fiber temperature $T_{n0} = 290$°C, initial gas temperature $T_{f0} = 20$°C, temperature of the channel wall $T_w = 20$°C; the changes in the value of Pr were due only to the heat conductivity of the medium.

It follows from the figures that the heat-exchange intensity decreases with an increase in Pr, and, as a result, the fiber temperature T_n (solid curves) at large values of Pr decreases more slowly than that at low values of Pr.

Another important distinction is that at small Pr the differences between the fiber temperatures at the center s and on the surface of the bundle p, and also between the temperatures of the medium (dashed curves) at the same points are rather appreciable. It decreases with an increase in Pr and is only a few degrees at Pr = 10 (in Figs. 3 and 4, the curves of the fiber temperatures at the center and at the boundary of the bundle practically merge).

Thus, for the given set of governing parameters, the smaller Pr, the less homogeneous the temperature fields of the medium and of the fiber bundle. The nonhomogeneity is associated with the influence of the wall whose temperature is considerably lower than the fiber temperature. As a result, at low values of Pr more intense heat transfer from the bundle occurs, which causes greater nonhomogeneity of the temperature field inside the bundle. Here it is interesting to note that the gas temperature in the peripheral regions of the bundle (Figs. 1 and 2) becomes even higher than the fiber temperature. Due to this, as is seen from Fig. 1 at Pr = 0.5, the fiber temperature on the surface of the bundle begins to rise again after intense cooling and after intersection of the curves of T_{ns} and T_{fs}.

In spite of these differences, the range of Nusselt-number variation is narrower than the wide range of Prandtl-number variation. Thus, at the ends of the range considered, Nu = 0.42 for Pr = 0.5 and 1, and Nu = 0.51 for Pr = 5 and 10. The variation of Nu numbers with the location of the fiber in the bundle is practically insignificant.

REFERENCES

1. H. Schlichting, *Boundary Layer Theory*, McGraw-Hill, New York (1968).
2. V. M. Yeroshenko and L. I. Zaichik, *Hydrodynamics and Heat and Mass Exchange at Penetrable Surfaces* [in Russian], Nauka, Moscow (1984).
3. V. I. Eliseev and Yu. P. Sovit, "Free convective heat exchange in open systems of vertical fibers," *Zh. Prikl. Mekh. Tekh. Fiz.*, No. 5, 88–94 (1990).
4. V. I. Eliseev and Yu. P. Sovit, "Heat exchange by mixed convection in moving bundles of fibers," *Inzh.-Fiz. Zh.*, **61**, No. 2, 230–239 (1991).

5. D. Happel and G. Brenner, *Small Reynolds Number Hydrodynamics* [Russian translation], Mir, Moscow (1976).
6. R. I. Nigmatulin, *Foundations of Mechanics of Heterogeneous Media* [in Russian], Nauka, Moscow (1978).
7. M. E. Shvets, "On approximate solution of some problems of boundary layer hydrodynamics," *Prikl. Mat. Mekh.* **13**, No. 3, 257–266 (1949).
8. L. G. Loitsyanskii, *Laminar Boundary Layer* [in Russian], Fizmatgiz, Moscow (1962).
9. A. Sh. Dorfman, *Heat Exchange in Flows Around Anisothermic Bodies* [in Russian], Mashinostroyeniye, Moscow (1982).
10. L. G. Loitsyanskii, *Fluid and Gas Mechanics* [in Russian], Nauka, Moscow (1970).
11. V. K. Migai and É. V. Firsova, *Heat Exchange and Drag of Tube Bundles* [in Russian], Nauka, Leningrad (1986).
12. V. I. Eliseyev and Yu. P. Sovit, "Hydrodynamics and heat exchange in a tube with a bundle of moving fibers," *Izv. Akad. Nauk Belarusi, Ser. Fiz.-Energ. Nauk*, No. 1, 76–83 (1992).